

Papers written by
Australian Maths
Software

SEMESTER ONE

YEAR 12

**MATHEMATICS SPECIALIST
REVISION 3
UNIT 3**

2016

**Section One
(Calculator-free)**

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work:

5 minutes

Working time for section:

50 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate

Standard items: pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: nil.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section One.

A formula sheet which may also be used for Section Two.

Structure of this examination

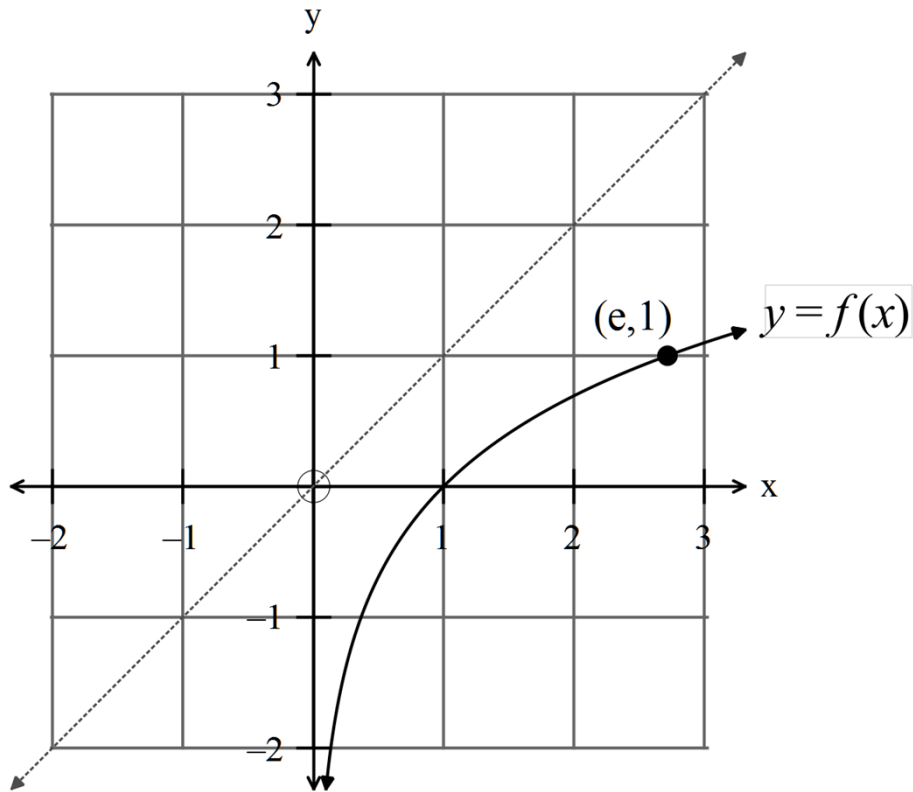
	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	5	5	50	50	35
Section Two Calculator—assumed	12	12	100	100	65
Total marks				150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answer in the Question/Answer booklet.
3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.
4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you **do not use pencil**, except in diagrams.
7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

1. (6 marks)

(a) The graph of $y = f(x)$ is drawn for $x > 0$ on the set of axes below.



(i) Sketch the inverse function $y = f^{-1}(x)$. (1)

(ii) Write down the equation of both functions, $y = f(x)$ and $y = f^{-1}(x)$. (2)

(iii) Write down the domain and the range of $y = f^{-1}(x)$. (2)

(iv) Given $f(e^2) = 2$, find $f^{-1}(2)$. (1)

2. (10 marks)

Matrices can be used to solve systems of equations such as

$$a_1x + b_1y + c_1z = d_1$$

$$a_2x + b_2y + c_2z = d_2$$

$$a_3x + b_3y + c_3z = d_3$$

where each equation represents a plane in space.

Solve each the following where possible and interpret the result.

$$(a) \left(\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 5 & 0 & 10 \\ 0 & 0 & 1 & -3 \end{array} \right) \quad (1)$$

$$(b) \left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{array} \right) \quad (2)$$

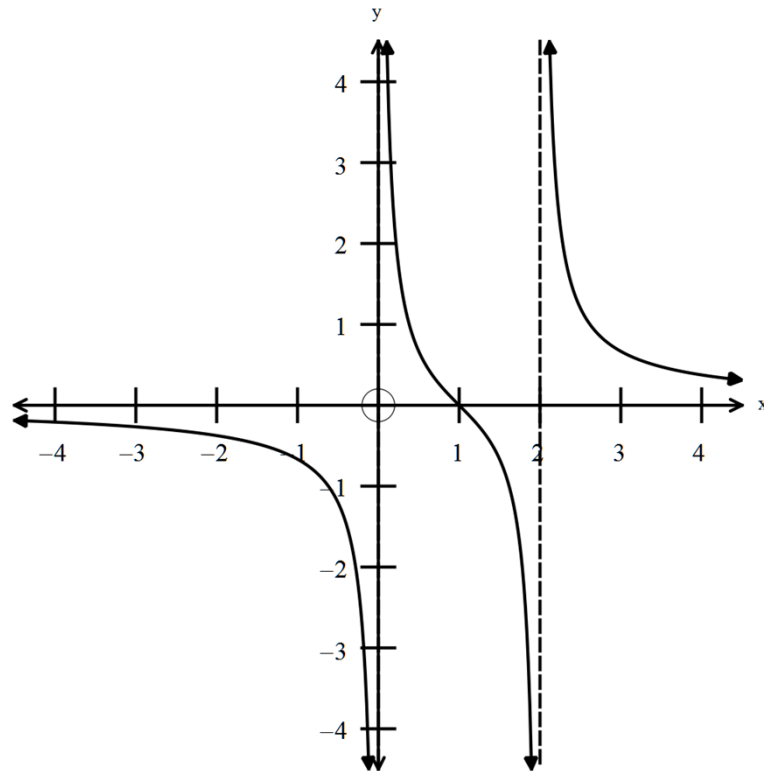
$$(c) \left(\begin{array}{ccc|c} 1 & 2 & -5 & -2 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad (2)$$

(d) Use Gaussian elimination to solve the set of equations

$$\begin{aligned}x + y + z &= 0 \\2x - y + 2z &= -6 \\2x - 2y + z &= -5\end{aligned}\tag{5}$$

3. (8 marks)

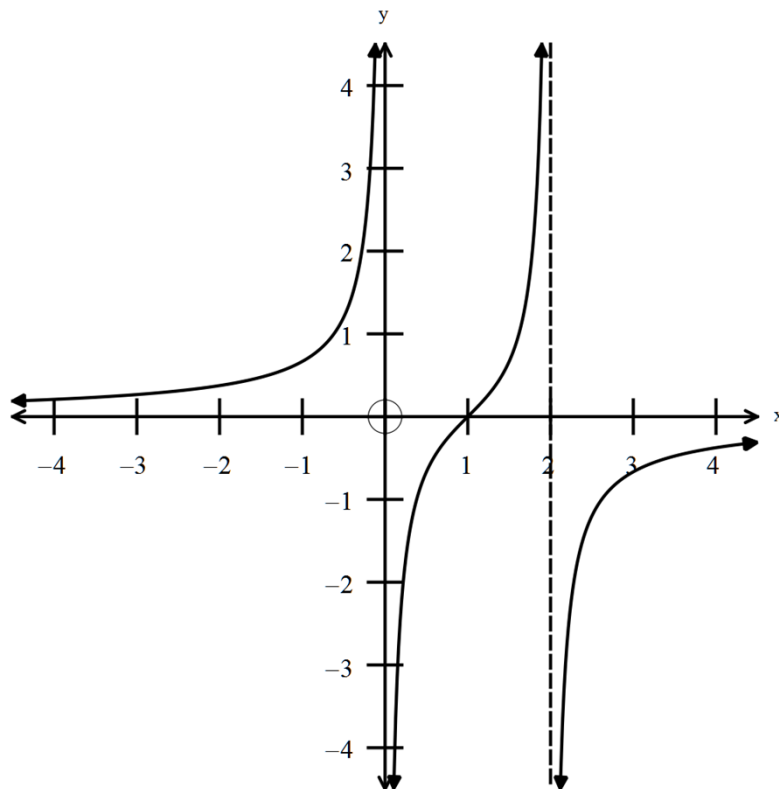
(a) Given the sketch of the function $f(x) = \frac{(x-1)}{x(x-2)}$



find the equations of the graphs below.

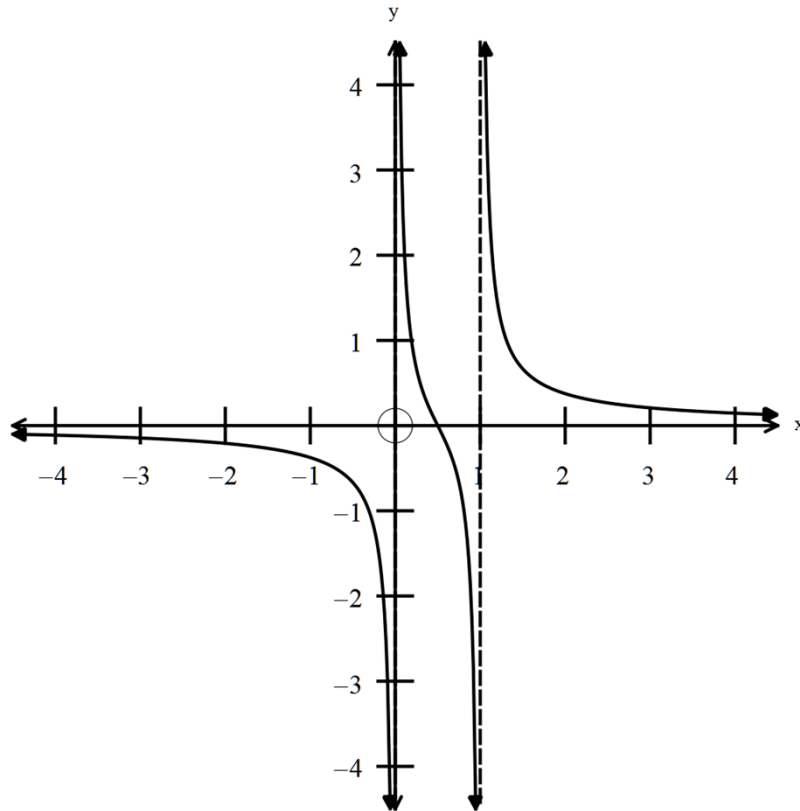
(i)

(2)



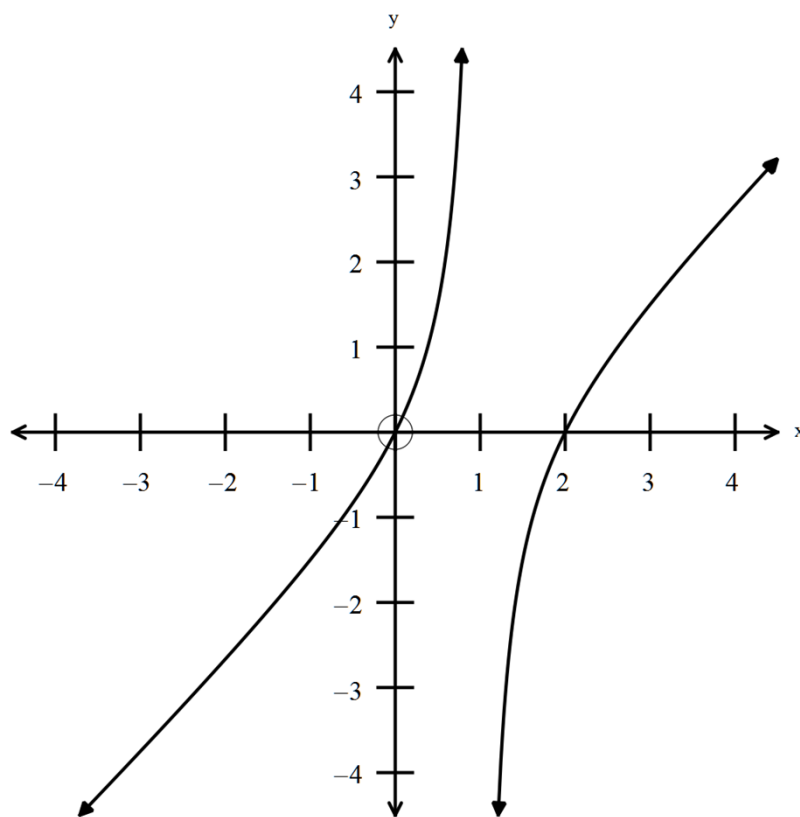
(ii)

(3)



(iii)

(3)



4. (13 marks)

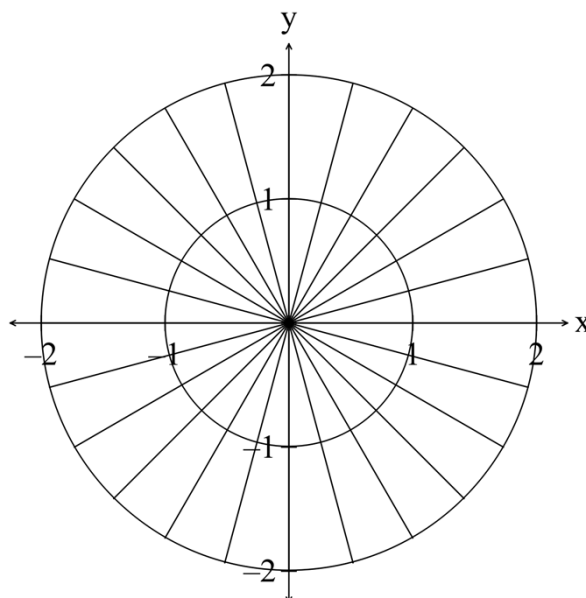
(a) Given $z^3 + az^2 + bz + c = 0$, find the values of a , b and c if the roots of the equation are 4 and $1 \pm 2i$. (3)

(b) Given one solution to the equation $z^3 + 2z^2 + 2z + 1 = 0$ is $z = -1$, find the other two solutions. (4)

(c) (i) Determine the solutions of the equation $z^3 = -8$ for $-\pi < \theta \leq \pi$. (3)

(ii) Explain the relationship between the roots. (1)

(iii) Illustrate the roots on the diagram below. (2)



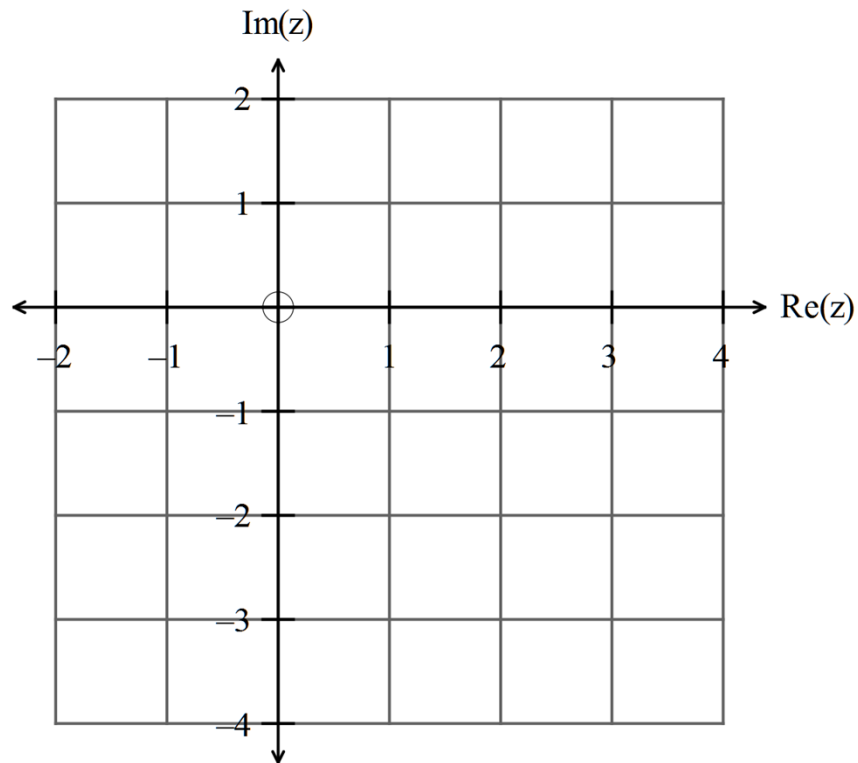
5. (13 marks)

(a) Simplify $(2-2i)^4$ (1)

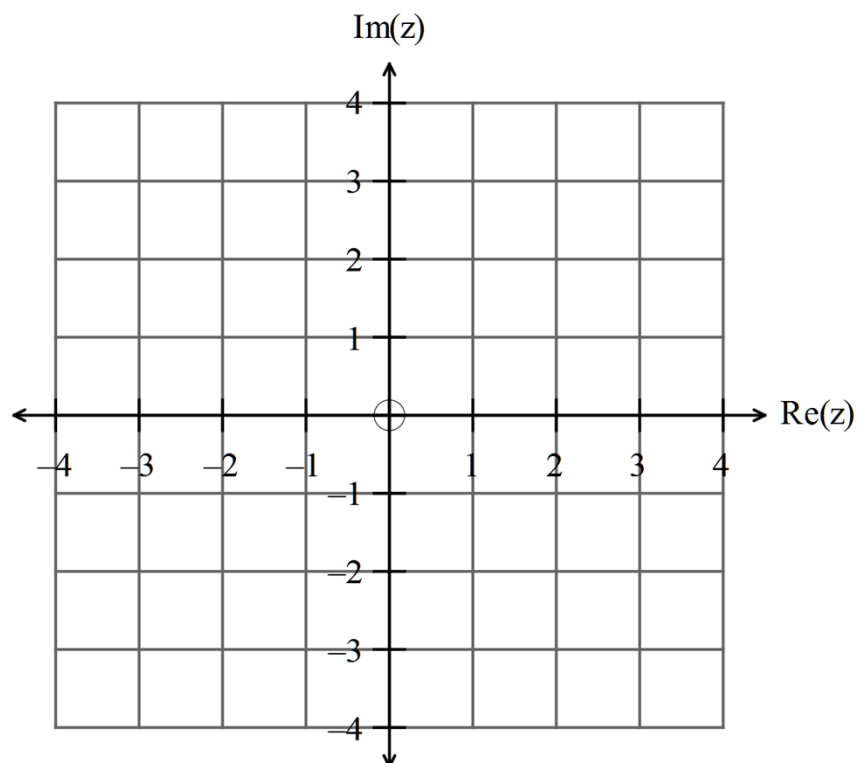
(b) Express $\frac{3-i}{3+i} - \frac{2+i}{1-i} + \frac{1}{i}$ with a rational denominator (3)

(c) Find real numbers x and y such that $(2+3i)(3-4i) = x + yi$. (2)

- (d) Sketch $\left\{ z : 1 < |z - 1 + i| \leq \sqrt{2} \cap -\frac{\pi}{4} < \arg(z) \leq \frac{3\pi}{4} \right\}$ on the set of axes below. (2)



- (e) Sketch $z_1 - z_2$ given $z_1 = 3 + 2i$ and $z_2 = 3i$ on the set of axes below. (2)



(f) If $z_1 = \operatorname{cis}\left(\frac{\pi}{4}\right)$, $z_2 = \operatorname{cis}\left(\frac{\pi}{3}\right)$ and $z_3 = 1 - i$

evaluate $\frac{(z_1)^4}{(z_2)^3} \times z_3$. (3)

END OF SECTION ONE